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A Comparison of Prices Generated by The Derivative Commodity Model (Ornstein-Uhlenbeck Process) With Those Obtained by The Conventional Arbitrage-Free Method of Pricing Forward Derivatives with Respect to Tea in Nduti Tea Factory Kenya.

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Abstract

Purpose: The purpose of this study was to compare the prices generated by the derivative commodity model with those obtained by the conventional arbitrage-free method of pricing forward derivatives with respect to tea.

Methodology: The study used descriptive survey research design. The study used descriptive survey research design. This study used secondary data which was collected from Nduti Tea Factory website. The target population of the study were 318 auction days auction days on the stock exchange spread over from 18/12/2007 to 2/12/2014. Purposive sampling was used to select 6 working days excluding Sundays and holidays starting from 18/12/2014 to 2/12/2014.Data from the websites was analysed using the Ornstein Uhlenbeck process, to derive descriptive results.

Results: The findings implied that there was variations in forward prices calculated by derivative commodity model as compared to those calculated conventional arbitrage-free.

Unique contribution to theory, practice and policy: The study provides need to insuring farmers from uncertainty by ensuring they get value for the input and costs of production. On the other hand, consumers are protected from the volatile food commodity prices. An incentive for the farmer is established and hence increased and more efficient productivity is witnessed. The study will lead to designing a simple commodity derivative with different times to expiry for tea in Kenya and elsewhere based on estimated future market prices. The results of this study will be of particular significance to farmers, cooperatives and general investors.

Keywords: *price derivatives, Ornstein Uhlenbeck process, commodity derivative, forwards, futures, options or swaps.*

1.1 INTRODUCTION

A commodity derivative contract exists where an individual acquires the right to buy or sell a commodity for a certain price on a specified date, at a margin price (price of the contract) from a contract seller who accepts this margin. These derivatives may take the form of forwards, futures, options or swaps.

Over the years gone by the use of the derivatives market has been a key figure in risk management under commodity trading. The use of forwards and futures dates back to the late 18th and early 19th centuries. The Chicago board of trade introduced exchange traded future contracts on cereals and agricultural products in 1848. This making it the world's first oldest futures and options exchange. The most common agricultural commodity derivatives traded at the Chicago board of trade include wheat, corn, maize and soybeans among others.

Commodity markets may either be in form of exchange traded or over the counter (OTC) trading, or both. Off-exchange trading, better known as over the counter (OTC) trading consists of contracts performed directly by two parties without external supervision.

The derivatives market was first introduced in a bid to mitigate the risk of farmers selling their produce below cost price. Commodity markets may either be in form of exchange traded or over the counter (OTC) trading, or both. Off-exchange trading, better known as over the counter (OTC) trading consists of contracts performed directly by two parties without external supervision. The main participants in the futures market includes the hedgers, speculators, and arbitragers. . Hedging through the use of derivatives is crucial in many sectors of the economy due to the changing and volatile nature of the world market. Arbitragers are investors who attempt to profit from price anomalies by trading in simultaneous transactions that offset each other and in the process acquiring risk-free profits. Speculators try to anticipate price movements with the hope of making a profit. Hedgers are viewed as risk averse while speculators are more prone to taking risks.

Trading commodities can be grouped into four major categories namely energy, metals, agricultural, livestock and meat. In the course of our project, we will deal with tea derivatives which are part of agricultural commodity derivatives

1.2 Problem Statement

Pests and diseases, climatic hazards (unreliable rainfall), delayed payments among other factors have led to price fluctuations in the tea industry. As a result, farmers are forced into settling for meagre proceeds and thus this weakens their morale in cultivation of the cash crops. Such farmers may tend to abandon tea in favour of other appealing ventures or seize farming altogether.

According to The Business Daily, the price of tea at the Mombasa auction hit a three month low towards the end of the year 2014. As a result, more than half a million farmers who sell their tea through Kenya Tea Development Agency (KTDA) were forced into earning lower bonuses. The tea board of Kenya, responsible for regulating the market, blamed unsold tea from previous auctions as having caused the price dip.

A system should be put in place so as to mitigate the risk presented by the fluctuating prices. The problems that this study aims to solve or at least control (to a considerable extent) include among others: Uncertainty- Control fluctuating prices by establishing a specified price over a period of

time, based on future expected spot prices of tea, deteriorating economic conditions- discourage manufacturers from importing tea while at the same time providing investment opportunities for local investors. Also demand supply deficit- give an incentive to tea farmers to produce and store more to meet demand, by assuring a fixed reasonable price and constant demand for produce. Finally, to enhance transparency- determine prices purely on the basis of demand and supply.

1.3 Study Objectives

- To compare the prices generated by the derivative prices model with those obtained by the conventional arbitrage-free method of pricing forward derivatives with respect to tea.

2.0 LITERATURE REVIEW

Blanco (2001) using Geometric Brownian motion model concluded that the speed at which prices revert to their long run levels may depend on several factors such as the nature, magnitude and direction of the price shock. If we calibrate the mean reversion rate for each month of the year using data exclusively from that month, we would find that for most markets the mean reversion rates differ considerably. The limitation on this model is that it has more complex processes that incorporate more information about the possible price changes, but at the cost of having to estimate many more parameters and increasing the probability of model errors.

Kristian, Miltersen, Eduardo and Schwartz (1999) developed a model to value options on commodity futures in the presence of stochastic interest rates as well as stochastic convenience yields. In the development of the model, they distinguished between forward and future convenience yields. Assuming normality of continuously compounded forward interest rates and convenience yields and log-normality of the spot price of the underlying commodity, they obtained closed-form solutions generalizing the Black-Scholes formulas. They provided numerical examples with realistic parameter values showing that both the effect of introducing stochastic convenience yields into the model and the effect of having a short time lag between the maturity of a European call option and the underlying futures contract have significant impact on the option prices.

According to no-arbitrage approach in Schwartz (1997) model, futures prices are determined by taking expectations of the future spot prices under an equivalent martingale measure. When establishing the dynamics of the basic state-variables under the equivalent martingale measure, it is assumed that risk premium is constant. In the present model, (log) futures prices are functions of the two stochastic state-variables.

Hull and White (1993) used the Vasi˘cek model to make the mean-reversion parameter under risk-neutral probabilities be a deterministic function of time. In their model they let the underlying commodity price be a geometric Brownian motion and also let the spot convenience yield have a Hull-White-alike structure but with all parameters time dependent, i.e. a generalized Ornstein-Uhlenbeck process. Their model is flexible thus making it possible to simultaneously match the structure of forward and futures prices, the current term structure of forward and futures volatilities and the inter-temporal pattern of the volatility of the forward and futures prices.

Brennan (1991) finds the empirical relationship between inventories of the commodity, spot prices, and convenience yields. When inventories are low, spot prices are relatively high, and convenience yields are also relatively high, since futures prices will not increase as much as the spot price, and vice versa, when inventories are high. Hence, there is empirical evidence of a consistent positive correlation between commodity prices and convenience yields for some commodities.

3.0 RESEARCH METHODOLOGY

The Simulation and Estimation of the Ornstein-Uhlenbeck Process and the Application of the Process to Commodities Markets and Modelling is a mean reversion process used extensively in finance to model interest rates and also by those who model commodities. The Ornstein and Uhlenbeck (1930) ('O-U') process, also referred to as the Vasicek (1997) process is the most popular model for such work. Mean reversion processes are generally attractive to model assets because they incorporate the economic argument that when prices are excessively high, demand reduces and supply increases which has a counterbalancing effect. When prices are low, the reverse occurs and prices plummet to some sort of long-term mean.

The Ornstein Uhlenbeck process, S, is modelled as follows:

$$ds = \lambda(\mu - S)dt + \sigma dW_t$$

Where;

σ, is the measure of volatility

λ, is the measure of speed of mean reversion

W_t, is the Brownian Motion, hence $W_t \sim N(0,t)$

μ, is the long-term mean to which the process tends to revert.

PRICING OF TEA DERIVATIVES

The Normal No- Arbitrage Pricing Equation

As we price these commodity derivatives, we make the assumption that the market is arbitrage free, meaning, if there aren't initial wealth and risk is not taken, the ultimate payoff should be null. Under this assumption, it can be laid out that the forward price, with time, T, to maturity of a commodity is actually given by:

$$f^T(t) = S(t)e^{r(T-t)}$$

Where, r, is the prevailing continuously compounded interest rate at time t. It is expected to remain constant over the period.

Pricing using the model

Using the MLE parameters for the realized model, in the above section, we price forwards with expiry date T under Q-martingale measure, according to Samir Masood (Sheihk, 2007), such that:

$$F(t,T) = \mathbb{E}_t^Q[P_T|B_t^1]$$

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(3.24)

The price Y is computed by calculating the expectation of e^{xt}. We let,

$$Y = e^{\alpha t} z_t$$

Substituting for dz_t from (3.2.7) and simplifying yields:

$$dY = d(e^{\alpha t} z_t) = \alpha e^{\alpha t} z_t + e^{\alpha t} dz_t$$

Integrating yields;

$$e^{\alpha T} z_T - e^{\alpha t} z_t = \alpha e^{\alpha t}(\mu - \frac{\sigma^2}{2\alpha})dt + \sigma e^{\alpha t} dW + e^{\alpha t} \ln J dN_t$$

$$z_T = (\mu - \frac{\sigma^2}{2\alpha}) + \left(z_t - \mu + \frac{\sigma^2}{2\alpha}\right) e^{-\alpha(T-t)} + \sigma \int_t^T e^{-\alpha(T-s)} dW_s + \int_t^T e^{-\alpha(T-s)} \ln J dN_s$$

Assumption:

The term C = $(\mu - \frac{\sigma^2}{2\alpha})$ takes into account the market risk.

Evaluating the expectation yields the following:

$$F(t,T) = \mathbb{E}_t[e^{z_T}|B_t]$$

$$= \mathbb{E}_t\left[e^{(\mu-\frac{\sigma^2}{2\alpha})} . e^{\left(z_t-\mu+\frac{\sigma^2}{2\alpha}\right)e^{-\alpha(T-t)}} . e^{\sigma\int_t^T e^{-\alpha(T-s)}dW_s} . e^{\int_t^T e^{-\alpha(T-s)}\ln J dN_s} | B_t\right]$$

$$F(t,T) = e^C \left(\frac{P(t)}{e^C}\right)^{e^{-\alpha(T-t)}} \mathbb{E}_t\left[e^{\sigma\int_t^T e^{-\alpha(T-s)dW_s}}|B_t\right] . \mathbb{E}_t\left[e^{\int_t^T e^{-\alpha(T-s)}\ln J dN_s}|B_t\right]$$

$$\mathbb{E}_t\left[e^{\sigma\int_t^T e^{-\alpha(T-s)dW_s}}|B_t\right] = e^{\frac{\sigma^2}{2}\int_t^T e^{-2\alpha(T-s)}ds}$$

$$\mathbb{E}_t\left[e^{\sigma\int_t^T e^{-\alpha(T-s)dW_s}}|B_t\right] = e^{\frac{\sigma^2}{4\alpha}[1-e^{-2\alpha(T-t)}]}$$

Thus, the forward price is obtained using the formula:

$$F(t,T) = e^C \left(\frac{P(t)}{e^C}\right)^{e^{-\alpha(T-t)}} e^{\frac{\sigma^2}{4\alpha}[1-e^{-2\alpha(T-t)}]+\int_t^T e^{(\mu g+\frac{\sigma^2}{2}g^2)-\lambda(T-t)}}$$

Where C is defined as previously mentioned.

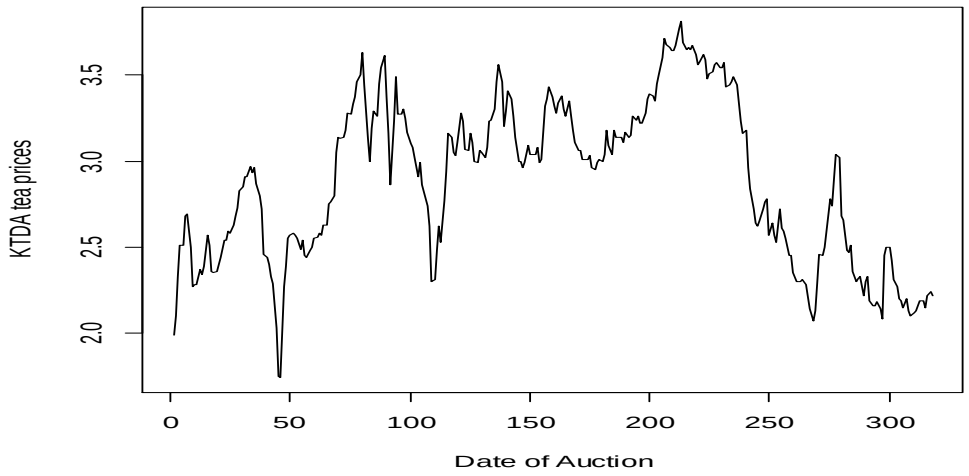
4.0 RESULTS AND DISCUSSIONS

4.1 Introduction

A graphical trend of the KTDA tea prices indicates the possibility that the process could be a random walk process. A random walk process is usually an AR(1) process. However, further tests are required inorder to confirm whether the data set follows an AR(1), AR(2) or an ARMA(1,1) process.

Steps in model building;

- Identify the kind of model (p,q) by running ACF and PACF. Use simulated models to compare the ACF and PACF
- Estimate the coefficients
- 3.Carry out diagnostics to prove that the residuals follow a white noise process. The various diagnostics for residuals include QQ plot, ljung box pierce statistics, rank test, sign test, AIC and BIC



Autocorrelation Function and Partial Autocorrelation Functions

The ACF of tea prices indicates that the process is non decaying, the PACF indicates 2 bars, a positive one and a negative 1.

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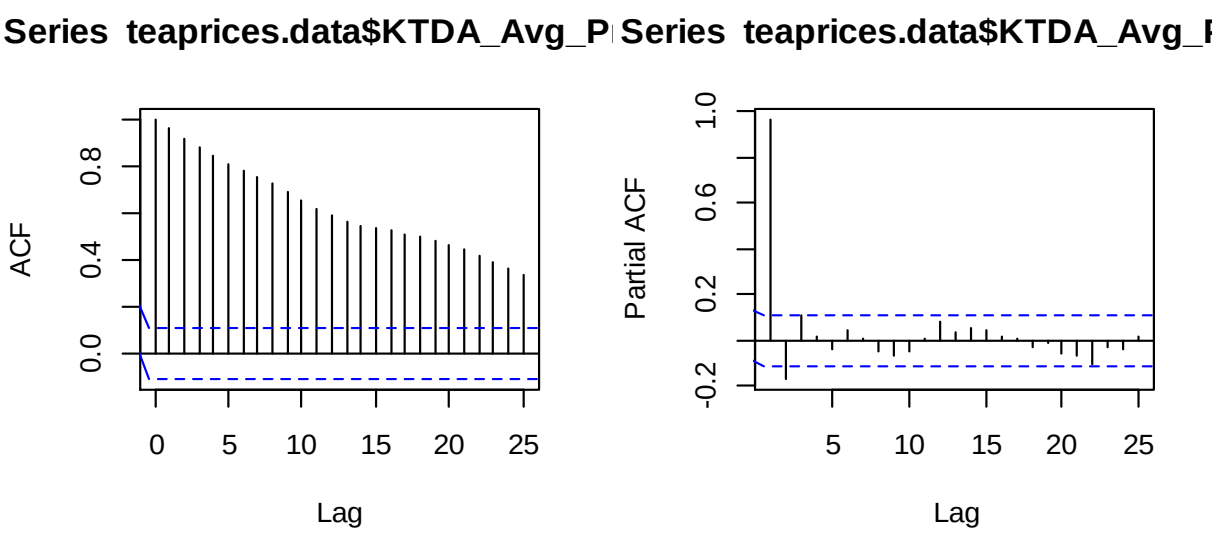
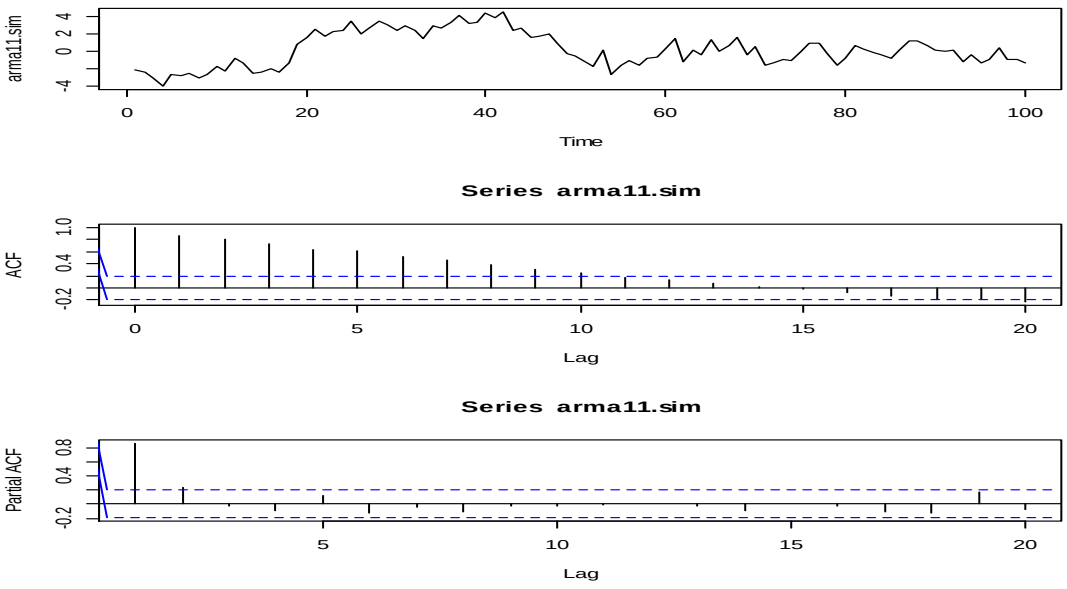


Simulations to enable model identification

AR(1) process: ACF and PACF



The non-decaying ACF is similar to the tea prices data set, the PACF has two significant spikes could be an indicator that this is an ARMA (1, 1) process.

Model Estimation (ARMA(1,1)

The ARMA(1,1) model can be estimated in MLE

$X_t = 0.9641X_{t-1} + 0.2955\ w_{t-1} + 2.7662 + w_t$

sigma^2 estimated as 0.01051

Table 1: Summary of Results

	Alpha(α)	Theta(θ)	Intercept/Mu(μ)	Sigma2(σ)
MLE	**0.9641**	**0.2955**	**2.7662**	**0.01051**

Residual diagnostics

The trend of the standardized residuals indicates that the residuals follow a white noise process. The ACF for residuals show that the covariance of residuals is zero and hence they follow a white noise process. The normal QQ plot indicates that the deviations from the line of best fit are small, hence no outliers and this is an indicator that the residuals follow a white noise process. The p values for L jung statistic indicate that the null hypothesis (white noise process) is not rejected and hence the residuals follow a white noise process

4.2 Pricing the Derivatives

Equations below was used to calculate the forward prices. This is referred to as the traditional approach

$f^T(t) = S(t)e^{r(T-t)}$

The forward prices based on the OUM model, that is the Vasicek model was calculated using the following equation

$$F(t,T) = e^{C}(P(t)/eC)^{e-\alpha(T-t)} \cdot e^{\sigma 2/4\,\sigma[1-e-2\,\sigma(T-t)]} + S_t \qquad (3.31)$$

Where C is defined as previously mentioned.

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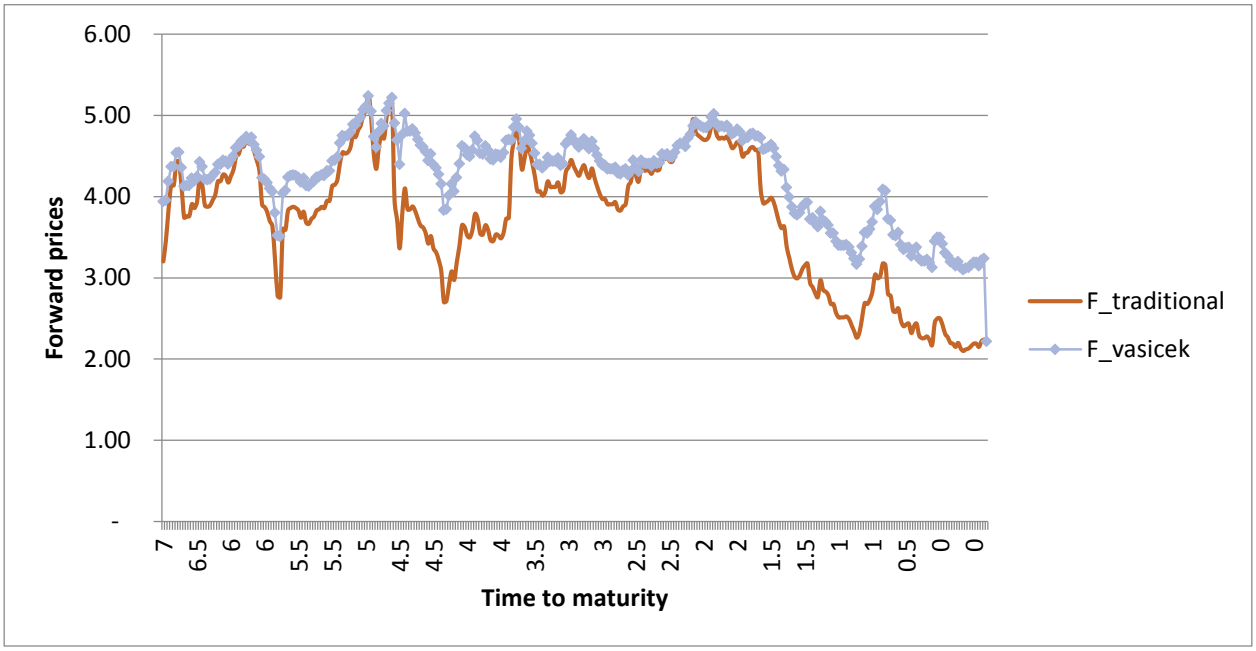

F Test

Table 2:correlation results

F-Test Two-Sample for Variances

	Variable 1	*Variable 2*
Mean	3.742843646	4.265003299
Variance	0.662564788	0.289725259
Observations	318	318
df	317	317
F	1.2	
P(F<=f) one-tail	0.14	
F Critical one-tail	2.29	

R code

```
require(astsa)
teaprices.data<-read.csv("teaprices.csv",header=TRUE)
teaprices.data
names(teaprices.data)
```

```
plot(teaprices.data$KTDA_Avg_Price,xlab="Date of Auction",ylab="KTDA tea prices",type="l")
par(mfrow=c(2,2))
acf(teaprices.data$KTDA_Avg_Price)
pacf(teaprices.data$KTDA_Avg_Price)
```

Simulations

```
## R command arim.sim
#AR(1)
par(mfrow=c(3,1))
ar1.sim<-arima.sim(model=list(ar=c(-.8)),n=100)
plot.ts(ar1.sim)
acf(ar1.sim)
pacf(ar1.sim)
##AR(2)
par(mfrow=c(3,1))
ar2.sim<-arima.sim(model=list(ar=c(-1.4,-.5)),n=100)
plot.ts(ar2.sim)
acf(ar2.sim)
pacf(ar2.sim)
#MA(1)
par(mfrow=c(3,1))
ma1.sim<-arima.sim(model=list(ma=c(-.9)),n=100)
plot(ma1.sim)
acf(ma1.sim)
pacf(ma1.sim)

# Simulate ARMA(1,1)
par(mfrow=c(3,1))
arma11.sim<-arima.sim(model=list(ar=c(0.9),ma=c(-0.2)),n=100)
plot(arma11.sim)
acf(arma11.sim)
pacf(arma11.sim)
```

#estimation by MLE

(model.mle=ar.mle(teaprices.data$KTDA_Avg_Price,order=2,demean=F,intercept=T))

sarima(teaprices.data$KTDA_Avg_Price,1,0,1**)**

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