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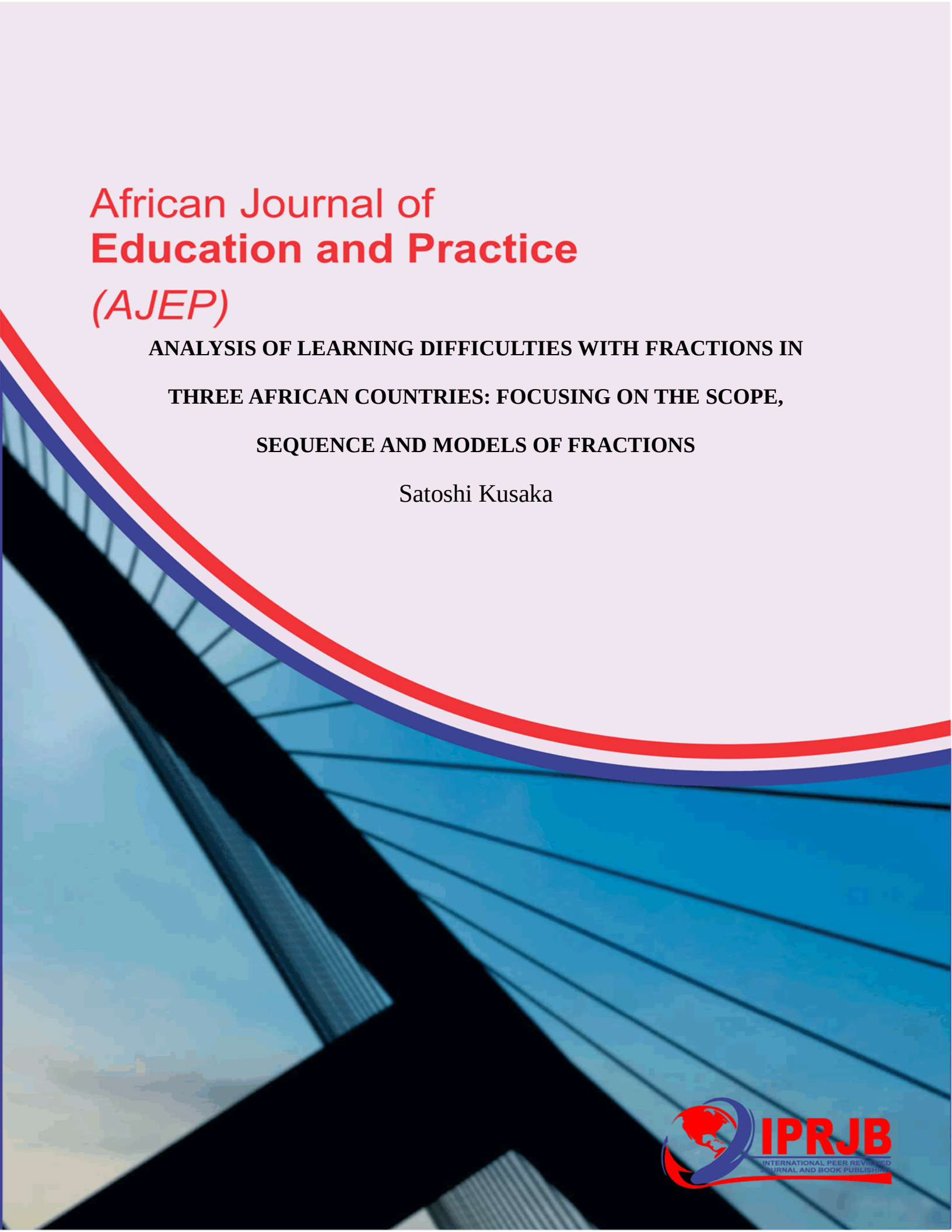

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ANALYSIS OF LEARNING DIFFICULTIES WITH FRACTIONS IN THREE AFRICAN COUNTRIES: FOCUSING ON THE SCOPE, SEQUENCE AND MODELS OF FRACTIONS

[1*]Satoshi Kusaka

Graduate School of Education: Naruto University of Education

*Corresponding Author's E-mail: kusaka@ksm.world

Abstract

Purpose: This study aims to compare and analyze learning content with regard to fractions, the order in which that content is taught in primary school mathematics curricula, and how it is presented in textbooks in three eastern and southern African countries, Zambia, Ethiopia, and Mozambique as well as to clarify the characteristics of the instruction concerning fraction in each of these countries.

Methodology: Firstly, we refer to the curriculum to extract information about the learning contents and their order in each grade. Secondly, concerning the meaning of fractions, we refer to the textbooks since we cannot clearly judge from the description in the curriculum. Thirdly, we focus on the common points and differences among the three countries and analyze the causes of difficulty in learning fractions.

Findings: There is a significant discrepancy between the grades in learning fractions among the three countries. In addition, the learning order differs to a certain degree. A common feature of the three countries regarding the order is the multiplication and division of fractions. For all three countries, while the addition and subtraction of fractions and types of fractions are handled separately by different grades, the multiplication and division are all taught in one grade. Further, how the meaning of fractions is taught is common to all three countries. In all the countries, the part-whole concept of fractions is mainly employed, and the fraction as measurement concept is not taught at all. Unfortunately, since children learn without considering fractions as measurements, their understanding of fractions will be limited.

Unique contribution to theory, practice, and policy: Regarding fraction, basic research on the teaching content and their order in African countries have not been conducted extensively. While improving the quality of education is a common goal globally, it is paramount to analyze the difficulty in learning fractions from the perspectives of the intended curriculum and textbook. The result will be the implication for revising the curriculum and suggestions for teaching fractions.

Keywords: *Fraction curriculum, Concept of fraction, Models for fraction, Mathematics in African countries*

1.0 INTRODUCTION

Understanding the meaning and computation of fractions is one of the most essential and difficult aspects of the "numbers and calculation" area of primary mathematics (Cramer et al., 2002; Siegler et al., 2013; Steffe & Olive, 2010; Hecht, Close & Santisi, 2003). In addition, Neagoy (2017) propounded that fractions are associated with a large part of what

students learn in school mathematics, and are an indispensable part of knowledge to understand ratio, proportion, percentage, etc., which are taught in the upper grades of primary school. Fractions are completely taught in primary education in most countries, but they also play a paramount role in secondary and higher education. The National Mathematics Advisory Panel (NMAP) of the US propounded that understanding algebra is essential in high-school and university mathematics, and the main reason students do not understand algebra is that they do not understand fractions, and they stressed that the goal of "fractions for all" should be addressed before setting the goal of "algebra for all" (NMAP, 2008).

Therefore, many researchers globally have pointed out the relevance and difficulty of teaching fractions in primary mathematics. Thus, fractions are one of the most essential teaching content in school mathematics. These observations are not limited to developed countries in Asia, Europe, and the US, but are extended also to African countries. In the international achievement survey of Grade 6 undertaken by the Union of Southern and Eastern African Consortium for Monitoring Educational Quality (SACMEQ), mathematics achievement is classified into eight levels, and fractions Level 3 specifies that "Translates graphical information into fractions," while Level 4 specifies that "Uses multiple different arithmetic operations on fractions." In SACMEQ III, conducted in 2007, more than half of the children were below Level 3 in 12 of the 16 participating countries. In other words, the students had not yet reached the level where they could represent fractions numerically. Therefore, in mathematics education, the difficulty in learning fractions is a common phenomenon globally.

Africa is said to be the last frontier on the planet. Its population is expected to keep increasing until 2100, which may lead to the opening up of various fields in the future. The quality of education in Africa is among the lowest globally, and improvement of academic ability is an urgent issue. However, there has been little basic research on the content and teaching of mathematics lessons in the curriculum that would guide national education. Goal 4 of the Sustainable Development Goals (SDGs), which focuses on education in African nations, is attracting international attention and interest while improving the quality of education, which is a common goal globally. Hence, what is taught concerning fractions and the order in which it is taught, as well as comparative analyses, are essential. How are fractions dealt in primary school mathematics curricula in African countries and presented in textbooks? This study aims to clarify what features and difficulties are involved in learning fractions in African countries through comparative analysis. The results of this study may offer suggestions for curricula and textbook revisions to improve the quality of education, which is the objective of Goal 4 of the SDGs.

2.0 LITERATURE REVIEW

2.1 Difficulties with fractions

There have been many studies on difficulties with fractions from the viewpoint of mathematics education (e.g., Cramer et al., 2009; Siegler et al., 2013; Neagoy, 2017). According to these, three major difficulties are associated with the learning of fractions—diversity in the meaning of the concept of fractions, diversity in the notation of fractions, and complexity of methods of calculation.

2.2 Diversity in the meaning of the concept of fractions

A fraction can mean many things, but fractions may be broadly divided into the following five types.

(1) Fraction as part-whole

A fraction that is based on an arbitrary quantity and divides a concrete object into several parts, regardless of size, and represents the number of parts of the whole. The part-whole concept of fraction is an effective starting point for constructing the meaning of fractions (Cramer & Whitney, 2010)

(2) Fraction as measurement

A fraction that indicates the magnitude of a quantity, such as 2/3m or 1/2g. It is distinct from the part-whole concept of fraction because it focuses on the magnitude of a quantity, rather than a fraction of a whole (Martinie, 2007).

(3) Fraction as operator

A fraction which functions as an operator is a fraction that represents a certain operation to obtain a number of parts of a certain thing, such as dividing "1" into three equal parts and taking two of them. The fraction as operator concept is paramount because it is related to other aspects of learning arithmetic (Johanning, 2008). However, Usiskin (2007) stated that curricula often do not adequately emphasize the fraction as operator concept.

(4) Fraction as division

Fractions which represent the quotients of integer divisions, such as $2 \div 3 = 2/3$. These fractions are even more meaningful in that the result of any division can be expressed as a fraction.

(5) Fraction as ratio

If there are two lengths, "a: 2m" and "b: 3m," and the answer to the question of how many times of b is a is that a is 2/3 times of b. Thus, this is a fraction representing the magnitude of a when b is 1. Kieren (1980) pointed out that rational numbers can be classified into five types of fractions—part-whole, measurements, quotients (divisions), operators, and ratios, and these five types cover all these interpretations. Thus, fractions have many meanings among rational numbers, and this is related to their difficulty. The order in which the meanings of these fractions should be learned is a critical issue from the viewpoint of mathematics education, and it differs among countries.

2.3 Diversity in the notation of fraction

The specialized and variable notation of fractions is another factor that makes their learning difficult. First, combining two numbers—the denominator and the numerator—as a single number can easily lead to confusion. There is a tendency to regard the denominator and numerator as independent numbers and compare them against one another (Cramer & Whitney, 2010). In addition, there are various other notations, such as proper fractions, mixed fractions, and improper fractions notations, and the procedure for interconverting between them is not simple. Furthermore, the infinite number of equivalent fractions, such as 1/2, 2/4, 4/8, etc., is also a factor that increases the difficulty.

2.4 Complexity of methods of calculations

Integers and decimals can be calculated based on the decimal notation system, but fractions cannot be calculated in the same way as integers and decimals. In the addition and subtraction of fractions with unlike denominators, the denominator should be made the same. In the addition and subtraction fractions where mixed fractions, improper fractions, and proper fractions are together, it may be necessary to perform conversions, or a fraction

may have to be dropped in the course of calculation. In multiplication and division, a reduction operation may need to be performed during the calculation. Further, in multiplication and division where mixed fractions, improper fractions, and proper fractions are together, it may be necessary to perform conversions, as in the addition and subtraction.

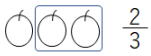
It is essential to understand the meaning of the calculation method to master the skills of calculating fractions. In particular, as regards the meaning of division, it is clear that when writing mathematical sentences and performing calculations, it is difficult for some children to understand the meaning. Students may well apply the algorithm, but they lack any conceptual understanding and have limited ideas of the sensibleness of their actions. One possible reason is the way fractions are presented in many textbooks—through meaningless procedures (English & Halford, 1995). Besides, when teaching the four arithmetic operations on fractions, teachers tend to focus on the methods of calculation and focus less on their meanings. Previous studies have shown that the reason for that is teachers themselves do not have adequate understanding of the meaning of the calculations, and it is difficult for them to explain using diagrams and number lines (Petit, Laird & Marsden, 2010; Vale and Davies, 2007; Lo and Luo, 2012). If children do not understand the meaning of the calculations, they have to rely on memorizing how to perform the calculations to solve problems. From this viewpoint, they come to feel the calculations of fractions is a stumbling block, and it makes them dislike mathematics.

2.5 Models for fraction

In learning the meaning and notation of fractions, it has been demonstrated that effective use of diagrams facilitates understanding (Cramer & Henry, 2002; Siebert & Gaskin, 2006). Areas, lengths, and sets of objects are mainly used as models to understand the concept of fractions (Petit, Laird, & Marsden, 2010). Since the concept of fractions shown by each model is different, different models have to be used depending on the concept to be taught. For example, a model obtained by dividing a circle is often used as an area model. Area models are suitable for teaching the part-whole concept (Cramer, Wyberg, & Leavitt, 2008). Length models are suitable for facilitating understanding of fractions as measurements, and previous studies have shown that they are the most suitable model for understanding fractions in terms of numbers (Petit, Laird, & Marsden, 2010; Siegler, Thompson & Schneider, 2011). Models using sets of objects are useful for understanding real-world fractions related to the concepts of fractions as division and ratio. Table 1 lists examples. Curricula and textbooks are required to present these three models according to their characteristics for children to understand the various meanings of fractions.

Table 1. Types of models for fraction and their characteristics

Types	Characteristics
Area models	 Area models are best suited for teaching fractions as part-whole.

Length models		Length models are suitable for understanding the fraction as measurement and fraction in terms of numbers.
Sets of objects		Sets of object models are useful for understanding real-world fractions related to division and ratio of fractions.

3. Objectives

This study aims to compare and analyze teaching content concerning fractions, the order in which that content is taught in primary school mathematics curricula, and how it is presented in textbooks in three eastern and southern African countries—Zambia, Ethiopia, and Mozambique—and to clarify the characteristics of the instruction concerning fraction in each of these countries.

4. METHODOLOGY

4.1 Reasons for selecting three countries for analysis

We selected Ethiopia, Zambia, and Mozambique as countries for comparative analysis. The reasons for choosing these three countries are the differences in their colonial history. School education in developing countries is often based on the system introduced by the ruling nation during the colonial era, and the mathematics curriculum is often introduced directly (Atweth & Clarkson 2001). Therefore, we selected the target countries based on differences between former ruling states during the colonial era. We decided to compare three neighboring countries with different colonial histories—Ethiopia was governed by Italy for five years after 1936, but has remained independent except for that period; Zambia was a British colony until her independence in 1964; Mozambique gained independence from Portugal in 1975. Table 2 shows the educational systems in each country.

Table 2. Overview of educational systems of the three target countries

	Ethiopia	Zambia	Mozambique
Primary	eight years	seven years	seven years
Secondary	four years	five years	five years
Period of compulsory education	eight years in primary education	seven years in primary education	seven years in primary education

4.2 Study methods

For achieving the objective of this study, we undertook a comparative analysis using the following methods.

(1) Comparative analysis of the teaching content of fractions and the order in which that content is taught in the curriculum

Focusing on teaching content and the order in each grade, we compared and analyzed the structure of teaching content in primary education and from the viewpoint of the meaning of fractions, and the order of their notations, shown in section 2.

(2) Analysis of how fractions are presented in textbooks

Focusing on model diagrams and examples of fractions shown in previous studies, we performed a comparative analysis of how fractions are presented in textbooks.

(3) Consideration of problems and difficulties in learning fractions

From the results of (1) and (2), we analyzed the commonalities and specific features of teaching fractions in each country, as well as the problems and causes of the difficulties.

4.3 Curricula and textbooks used

In this study, we analyzed the latest curriculum and textbooks approved by the Ministry of Education of each country. Ethiopia uses a curriculum that was revised in 2009 (Ministry of Education 2009) and Al Ghurair's textbook of mathematics, which is designated as a national textbook. Zambia uses a curriculum that was revised in 2013 (Curriculum Development Centre 2013), and as a textbook, Oxford's "Let's do Mathematics," which was approved by the Ministry of Education of Zambia in 2016. Mozambique uses a curriculum revised in 2015 (Ministério da Educação 2015). Regarding textbooks, national textbooks are currently being developed in line with the revision of the curriculum, but since textbooks for upper grades of primary education have not been issued, "Descobrir a Mathematica 5" from Plural Editores, published in 2014, is used.

5. Results and discussion

5.1. Teaching contents and their order

Table 3 shows the teaching contents of fractions, the order in which the contents are taught, and the types of fractions handled in the three countries. Since this study aims to compare teaching contents and their order, we extracted only the contents from the curriculum. In addition, we described the types of fractions handled with reference to textbooks if they were not described in the curricula.

As regards school grades that learn fractions, Ethiopia has five years, 1st to 5th grade; Zambia has five years, 3rd to 7th grade; and Mozambique has four years, 4th to 7th grade. Fractions are taught for four or five years in all three countries, but the starting academic year varies significantly among the countries. There are various reasons for this, but a more detailed analysis should probably be performed with respect to cultural backgrounds, such as fraction culture and decimal culture, as well as other teaching contents in the same area, such as integers and decimal numbers. The order in which the content is taught also differs significantly among countries.

In Ethiopia, students learn fractions for five years—from the 1st to 5th grade—but learn the concept, types, and equivalent fractions over the first three years, then learn the four

arithmetic operations on fractions in the 4th and 5th grades. As for the four arithmetic operations on fractions, 4th graders learn only the addition and subtraction of fractions with like denominators. Operations other than the addition and subtraction of fractions with like denominators, such as the addition and subtraction of fractions with unlike denominators, and multiplication and division of fractions, are all learned in the 5th grade. To learn the multiplication and division, which are the most difficult calculations to understand together with the addition and subtraction of fractions with unlike denominators within one year obviously places a tremendous burden on children. As for the meaning of fractions, when fractions are introduced, the part-whole concept of fraction, which is the most effective for understanding the meaning of fraction, is taught. However, in comparing the sizes of fractions after they are introduced, and learning types of fractions, such as unit fractions and equivalent fractions, everything is described in terms of this concept. Furthermore, all explanations for calculating fractions, starting from the 4th grade, are made using this concept.

In Zambia, students learn fractions for five years—from the 3rd to 7th grade. Focusing on teaching content, 3rd graders learn the basic concept of fractions and then learn the addition and subtraction of fractions with like denominators. In the 4th grade, improper fractions and mixed fractions are introduced, and in the same grade, they are used to learn the addition and subtraction of fractions with like denominators. In the 5th grade, children learn the addition and subtraction of fractions with unlike denominators, including improper fractions and mixed fractions. In the 6th grade, they learn the multiplication and division of fractions. By the 6th grade, the subject of fractions has been studied in its entirety, and the content of 7th graders can be described as a review of the four arithmetic operations on fractions. Thus, the types of fractions and the learning of addition and subtraction are linked to form a spiral arrangement. From the viewpoint of mastering the content, it is effective for 7th graders that there is time to re-learn all the teaching contents of fractions. However, focusing on the meaning of fractions, as in Ethiopia, although a small amount of activity related to fraction operators is included, fundamentally, everything from the introduction of fractions to the four arithmetic operations is explained in terms of part-whole concept, which leads to the teaching of fractions as numbers.

Mozambique is the slowest of the three countries in learning fractions, which does not start until the 4th grade. 4th graders learn the concept of fractions, 5th graders learn the addition and subtraction of fractions with like denominators, 6th graders learn the addition and subtraction of fractions with unlike denominators, and 7th graders learn the multiplication and division of fractions. The treatment of the meaning of fractions is very different from that of the other two countries. The fraction as division concept is used to introduce fractions. In addition, the fact that the fraction as ratio concept is handled in primary education (6th and 7th grades) is also distinct from the other countries.

What is common among the three countries in regards to the teaching of fractions is the order of the four arithmetic operations on fractions. Concerning the addition and subtraction of fractions, learning grades are divided according to the difference between fractions with like and unlike denominators, and types of fractions, such as improper and mixed fractions, are dealt with carefully over two or three years. Meanwhile, in the case of multiplication and division, all types of multiplication and division are treated in one grade without distinguishing whether multipliers and divisors are integers or fractions. Focusing on the meaning of calculation using the properties and rules of fractions, there is a significant difference depending on whether the multiplier is an integer or a fraction. Based on the idea of calculation when the multiplier is an integer, the meaning of multiplication

when the multiplier is a fraction can be understood, so it should be handled carefully in different grades. In addition, previous studies have shown that in the division, when the divisor is a fraction, it causes extreme difficulty in making mathematical sentence. However, focusing solely on the method of calculation, the difference between multipliers and divisors, whether integers or fractions, is not as significant as the addition and subtraction of fractions with like and unlike denominators. In other words, the focus is on mastering methods of calculation rather than understanding the meaning of fractions, so it may be understood as a way of teaching where everything is learnt together in the same grade.

What is common among the three countries as regards the meaning of fractions is that the part-whole concept is mainly used for teaching everything about fractions, such as the meaning of fractions, types of fractions, and arithmetic operations on fractions. Moreover, one of the major features common to the three countries is that fraction as measurement are not taught at all. Although the curriculum includes descriptions of the part-whole concept and fraction as operator and ratio concepts, there is no description of the fraction as measurement concept. In some textbooks, questions concerning the fraction as measurement concept are given as text problems, but they are not consciously treated as example problems. For the part-whole concept, the reference quantity is arbitrary, but the fraction as measurement are fractions whose reference quantities are expressed in universal units such as 1L and 1m, and it has been pointed out that as there is a large gap in the understanding of these types, this is a critical issue in formulating a meaning of fractions. In all three countries, after fractions are introduced by the part-whole concept, there is almost no effort to tackle the fraction as measurement concept, and learning progresses to fractions as numbers. Fractions are also compared in arbitrary units based on the part-whole concept and not in universal units, so children would have difficulty in understanding the meaning of fractions.

A typical example that arises from the lack of understanding of the fraction as measurement concept is a problem that was raised in a national scholastic ability survey for 6th graders in Japan in 2010. In the question, “If you divide 2 liters of juice into three equal parts, how many liters will one part have? Let us write the answer in fraction,” the typical mistake was “1/3 liter” or “1/3.” Another example was, a 1m paper tape and a paper tape of arbitrary length are given out, and the latter is used to make 1/2m. It was reported that when the paper tape of arbitrary length is divided into two equal parts without using the 1m tape, some children thought that the length of one of them would be “1/2m.” This type of mistake has also been experienced in the problem of associating fractions with positions on a number line and was reported to be “an incorrect answer due to a part-whole fraction model” (Novillis-Larson 1980; Kerslake 1986). Hence, to teach children that a half of a length is different when the original length is different, it is paramount to make them aware of the original “quantity,” and consider it as a fraction. In other words, if teaching that concept is omitted, learning proceeds without the idea of quantity being linked to fractions, and as a result, fractions are limited to a very narrow meaning. From an analysis of curricula and textbooks from the three countries to date, the fact that the main purpose is to be able to calculate fractions rather than to understand their meaning is conjectured to be one reason the fraction as measurement concept is not given sufficient importance. In view of previous studies, it is clear that the negligence of the fraction as measurement concept is a major hindrance to the understanding of fractions in general.

The facts that the spiral nature of the learning of multiplication and division of fractions is not taken into account, that they are studied together in one grade, and that the presentation

of the meaning of fractions is largely biased and narrowly limited to the part-whole concept, can be said to be one of the major causes of difficulty with fractions in the three countries.

Table 3. Arrangement of learning contents and types of fractions

	Ethiopia	Zambia	Mozambique
1	**(Part-whole)** Meaning of halves and quarters and three quarters by cutting objects		
2	**(Part-whole)** - Relationship of "half," "quarters," and "three quarters." How to write "half," "quarte and and "three quarters" numerals.		
3	**(Part-whole & Division)** - Unit fractions from 1/2 to 1/10. - Divide numbers in half or quarter Comparison of easy fraction		
4	**(Part-whole & Division)** - Fractions as how many pieces of whole pieces - Comparison of fractions with same denominators - Finding equivalent fraction by calculation Addition and subtraction of fraction with like denominator	**(Part-whole & Operator)** - Proper fractions - Showing fractions by figure - Addition and subtraction of fractions with like denominators	**(Part-whole &Division)** - Meaning of fractions - Fraction with denominator 10 and numerator 1 to 10 - Comparison of fraction with like denominator
5	**(Part-whole & Division)** - Proper, improper, and mixed fractions. - Comparison of fractions with unlike denominators - Addition and subtraction of	**(Part-whole and Operator)** - Equivalent fraction - Mixed and improper fractions - Addition and	**(Part-whole, Operator & Division)** - Indicate the number of minutes shown in the figure. - Comparison of

	fraction with like and unlike denominators - Multiplication and division of fractions	subtraction of proper, improper and mixed fractions with like denominator	fraction with like denominator - Addition and subtraction of fraction with like denominator
6		**(Part-whole, Operator & Division)** - Multiplication and division of fractions	**(Part-whole, division & Ratio)** - Reading and writing fractions - Mixed and improper fractions - Equivalent fraction
7		**(Part-whole, Operator & Division)** - Four operations on fraction	**(Part-whole, division & Ratio)** - Addition and subtraction of fractions with unlike denominators - Multiplication and division of fractions,

5.2. Model diagrams of fractions used in textbooks

We now discuss, in detail, specific parts of the presentation of fractions and the model diagrams used in textbooks of the three countries.

Area models

When fractions are introduced, Zambia and Ethiopia start with the part-whole concept, and the area models are used. In Mozambique, fractions are introduced using the fraction as division concept, and the explanation is accompanied by the area models. The common point among the three countries is that the area models are used not only to introduce fractions but also throughout the learning of fractions up to the final four arithmetic operations. This is probably because in all three countries, as mentioned earlier, since it is mainly the part-whole concept which is dealt with from the introduction up to the four arithmetic operations, the area models are most suitable for supplementary explanation.

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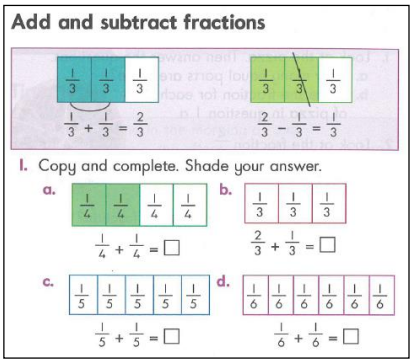

Figure 1. Addition of fraction (5th grade, Zambia)

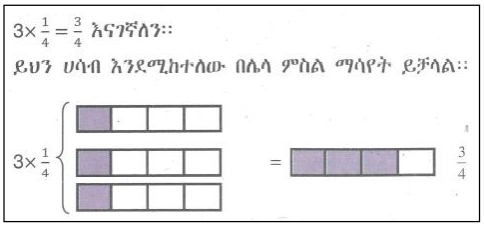

Figure 2. Multiplication of fractions (5th grade, Ethiopia)

Length models

In all three countries, no length model is used as model diagrams. However, in Mozambique and Zambia, textbooks use line segment diagrams (tape diagrams). In Mozambique, they are used to compare fractions with unlike denominators (Figure 3). The whole line diagram is 1, and the part-whole concept is shown using the line segment diagrams. Because this is a comparison of the magnitudes of fractions, it is necessary to go through the step of understanding fractions as numbers or quantities. Therefore, comparing magnitudes with only an understanding of the part-whole concept would probably confuse children. In Zambia, line segment diagrams (tape diagrams) are used for learning equivalent fractions. Again, they are presented as the part-whole concept rather than the fraction as measurement concept. Using a tape diagram that suggests a rightward extension, such as a number line, facilitates understanding of the fraction as measurement concept. In the discussion of curriculum analysis and use of the area models in the previous chapter, we stated that learning fractions in the very narrow sense of only the part-whole concept is a major factor in children's lack of understanding in the whole approach to fractions. This is again suggested by the fact that number lines are not used as model diagrams in textbooks.

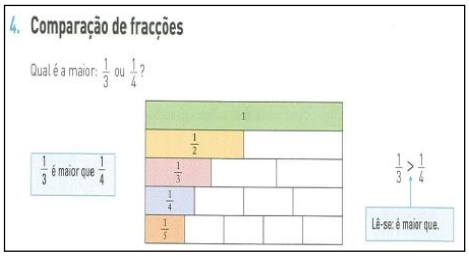

Figure 3. Comparison of fractions with different denominator sizes

(5th grade, Mozambique)

Sets of objects

Mozambique uses a model diagram of a set of objects when fractions are introduced. In Mozambique, fractions are introduced using the fraction as division concept, so a set of objects that is most suitable for understanding divisions is used (Figure 4). In Ethiopia too, diagrams of sets of objects are used to introduce divisions in the 3rd grade (Figure 5). Hence, in both countries, diagrams that take advantage of the characteristics of fractions are used. Meanwhile, in Zambia, when fractions are introduced, exercises using model diagrams of sets of objects are presented (Figure 6). For A and B, one fruit can be regarded as the whole, but taking the example of a cake C, it is difficult to interpret the whole as one, so this cannot be considered a suitable problem to give children immediately after introducing the part-whole concept.

As regards model diagrams of fractions used in textbooks, it became clear that area models were frequently used in all three countries, while number lines were not used at all. This is strongly related to the fact that children learn only with the part-whole concept. Further, in Mozambique and Ethiopia, when introducing the fraction as division concept, diagrams of sets of objects are used, so the diagrams are suitable for the meaning of fractions.

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Figure 4. Introduction of fractions

(4th grade, Mozambique)

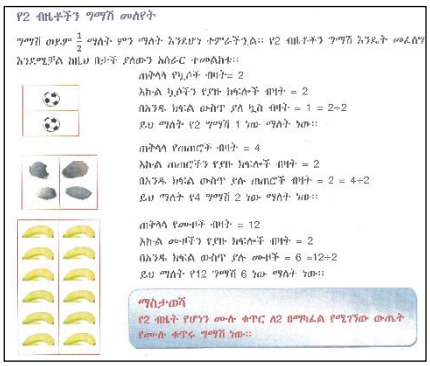

Figure 5. Fraction of quotients

(3rd grade, Ethiopia)

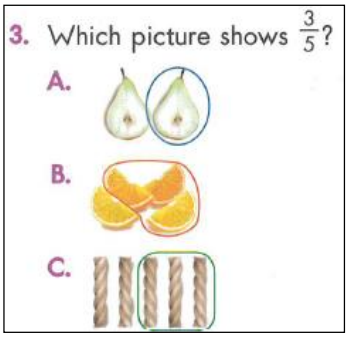

Figure 6. Exercises after the introduction of fractions

(3rd grade, Zambia)

6. Conclusion

In this paper, we focused on three countries in eastern and southern Africa concerning the difficulty in learning fractions, which is a common global perception in mathematics education, performed a comparative analysis of teaching content and order, and attempted to clarify one aspect of the difficulty.

As a result of the analysis, we found that there was a great discrepancy between the grades in learning fractions and that the order in which the content was taught also differed to a certain degree. A common feature among the three countries regarding the order of teaching content is how fractional multiplication and division are handled. For all three countries, while the addition and subtraction of fractions and types of fractions are handled separately by different school grades, the multiplication and division are all taught in one grade.

Further, how the meaning of fractions is taught is common to all three countries. In all the countries, the part-whole concept is mainly used, and the fraction as measurement concept is not taught at all. Unfortunately, since children learn without considering fractions as measurements, their understanding of fractions is limited.

Poor arrangement of the learning content of multiplication and division of fractions and negligence of the fraction as measurement concept can be said to be one of the major causes of difficulty in learning fractions. The reason the teaching of fractions is set out and arranged in this way may be that emphasis is placed on being able to calculate rather than to understand the meaning of fractions.

This study revealed that, although school grades and the teaching order of fractions differed to a certain extent, the cause of the difficulty in learning fractions was common to all three countries. Although the study only considered three countries in eastern and southern Africa, the accumulation of such basic research has implications for improving the true quality of education, which is the aim of Goal 4 of the SDGs. We hope that similar research will be conducted in other countries.

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